RESEARCH MEMORANDUM

STATUS OF FLUTTER OF FLAT AND CURVED PANELS

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NATIONAL ADVISORY COMMITTEE
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SUMMARY

Representative results are presented to show the current status of the panel flutter problem. The discussion includes flat panels with and without midplane stresses, buckled panels, and both unstiffened and stiffened infinitely long circular cylinders.

INTRODUCTION

The flutter of the skin panels of supersonic airplanes and missiles has aroused considerable interest in recent years. Numerous investigations (refs. 1 to 28) have determined that panel flutter may govern the design of at least very thin panels such as those used in fairings and in radiation shielding. Representative results of some of these investigations, which tend to illustrate the current status of the problem, are presented in this report. The discussion is divided roughly into three parts: First, the flutter of flat panels; second, the flutter of panels stressed beyond the buckling load; and third, the flutter of thin-walled circular cylinders.

SYMBOLS

$$A_p = \frac{P_x a^2}{D} + 2\left(\frac{a}{b}\right)^2 I_{pp}^{(2)}$$

a panel dimension in chordwise direction

$$B_p = \left(\frac{a}{b}\right)^2 I_{pp}^{(2)}\left[\frac{P_y a^2}{D} + \left(\frac{a}{b}\right)^2 \frac{I_{pp}^{(4)}}{I_{pp}^{(2)}}\right]$$

b panel dimension in spanwise direction

D panel stiffness, $\dfrac{Et^3}{12\left(1 - \mu^2\right)}$

E Young's modulus

$I_{mj}^{(i)} = \int_0^1 \phi_m(\xi)\frac{d^i}{d\xi^i}\left[\phi_j(\xi)\right]d\xi$

i,j,m,p integers

M Mach number

n integer, number of circumferential waves around the cylinder

P_x chordwise midplane load per unit spanwise length

P_y spanwise midplane load per unit chordwise length

q dynamic pressure

$R = \frac{P_y}{P_x}$

r cylinder radius

t thickness

w panel deflection

x chordwise coordinate

y spanwise coordinate

λ dynamic-pressure parameter, $\frac{2qa^3}{\sqrt{M^2 - 1}\,D}$

μ Poisson's ratio (taken equal to 1/3 throughout)

$\phi_m(\xi)$ mth mode of uniform clamped-clamped beam of unit length

ξ dimensionless coordinate

$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$

FLAT PANELS

Up to the present time, the largest research effort has been devoted to theoretical study of flat panels. Typical analytical results for flat panels are shown in figures 1 and 2.

The results in figure 1 apply to panels of infinite aspect ratio, that is, panels which extend to infinity in the spanwise direction and, hence, behave structurally like a beam. Figure 2 contains results for a panel with finite aspect ratio. Both plots give the panel thickness ratio t/a necessary to prevent flutter, at various Mach numbers, of steel panels at sea level. The results are restricted to panels which are unstressed; that is, they have no midplane tension or compression.

In both figure 1 and figure 2, the dashed curves at low supersonic Mach numbers are two-mode results based on the use of unsteady linearized air forces. (See refs. 8, 10, and 20.) The solid curves at high Mach numbers are two-mode results obtained by using much simpler "static" air forces in which no account is taken of unsteady effects. (See refs. 22, 26, and 28.) Experience has shown that, for the high Mach numbers, the static approximation yields results in close agreement with those yielded by more refined theories. The smooth merging of the dashed and solid lines for each configuration illustrates this close agreement.

In figure 1, curves are shown for three different configurations: an array of panels continuous over equally spaced supports, a single panel with pinned ends, and a single panel with clamped ends. The general effect of panel boundary conditions may be seen from the relative positions of the curves at high Mach numbers. The most critical configuration, requiring thicker panels to prevent flutter, is the array of panels on equally spaced supports.

The effect of aspect ratio on required panel thickness is shown in figure 2. In this case, both curves are for a single panel with pinned edges. The upper curve applies to an infinite span panel and is the same as the middle curve of figure 1. The lower curve applies to a square panel. As would be expected, reducing the aspect ratio from infinity to one reduces the required thickness.

It should be pointed out that the moderate reduction in thickness ratio at high Mach numbers is due primarily to the structural effects of aspect ratio. (See ref. 26.) The aerodynamic effect is less than 2 percent throughout the range of the solid curves. Thus, the use of aerodynamic strip theory would give accurate boundaries for the square plate for these high Mach numbers.

The situation is apparently different for the low Mach numbers. In addition to the moderate structural effect, there is now a large aerodynamic effect of aspect ratio and a resulting large reduction in thickness in changing from the infinite-aspect-ratio panel to the square panel. It is interesting to note that, although the low supersonic Mach number range appears to be critical for infinite-aspect-ratio panels, this may not be the case for the simply supported square panel for which the results show a steady increase in thickness with increase of Mach number.

It should be pointed out that the dashed curves at the low Mach numbers in figures 1 and 2 are based on a rather limited number of calculations and this region has not yet been carefully explored. It is known, however, that structural damping is fairly effective in reducing the large thicknesses required for infinite-aspect-ratio panels. (See ref. 10.)

It is also worthwhile to note that a small amount of experimental data has been reported at Mach numbers of 1.3 and 1.56 for clamped panels which behave like the infinite-aspect-ratio panel. (See refs. 6 and 13.) The data are not shown in figure 1 because they apply to a different altitude and material. When compared with calculations made on the same basis, experiment and theory agree at Mach number 1.56. At Mach number 1.3, the data show an increase of thickness but not the large increase predicted by the theory. The difference may be due in part to structural damping.

So far, only panels with no midplane stress have been considered. However, the effect of midplane tension or compression stresses on the required thickness of flat panels has been investigated and is found to be important (ref. 26). This effect is illustrated by the results shown in figure 3.

Figure 3 is a plot of a modified-thickness-ratio parameter $\left(\sqrt{M^2 - 1}\,\frac{E}{q}\right)^{1/3}\frac{t}{a}$, containing both panel and air properties, against a chordwise compression parameter $\frac{P_x}{(\text{Buckling } P_x)_{P_y=0}}$. The coefficient of the thickness ratio contains the Mach number M, Young's modulus E, and the dynamic pressure q and allows the application of the plotted results to all Mach numbers above about 1.5. The compression parameter, which specifies stress in the chordwise direction, is the ratio of chordwise compression P_x to the buckling value of chordwise stress which corresponds to zero stress in the spanwise direction. Positive values of this parameter indicate chordwise compression and negative values imply tension. Curves are shown for two pinned-edge panels with aspect ratios of 0.5 and 1.

It must be remembered that the calculations apply to flat panels only. Hence, the curves in figure 3 are valid only up to the chordwise load which results in buckling. If the midplane stress in the spanwise direction is zero, the curves are therefore valid, by definition, only up to the compression parameter equal to one. If tension is applied to the panel in the spanwise direction, the chordwise load necessary to cause buckling is raised; the curves would then be valid to a higher value of the compression parameter. Spanwise compression would have the opposite effect. It should be noted that this is the only influence of spanwise stress on the flutter boundaries of flat panels. (See ref. 26.)

The influence of chordwise stress is shown by the variation of the curves themselves. Note especially that, for each aspect ratio, there is a critical value of chordwise compression for which the theory requires very large thicknesses to prevent flutter. In real panels, the required thickness in this range is dependent on the amount of structural damping. However, it is apparent that these critical combinations of aspect ratio and chordwise compression should be avoided in design.

The mode shapes of flutter of flat panels have an interesting feature that is illustrated in figure 4. Calculated chordwise variations of typical flutter-mode shapes are shown for panels with both pinned and clamped edges. The air flow is from left to right. Note that the motion is concentrated toward the rear of the panel for both panels. This "tail-wagging" type of motion is characteristic of the observed motion in actual instances of panel flutter.

BUCKLED PANELS

Consideration will now be given to the flutter of panels which have been buckled by heating or by the application of midplane loads. Buckled panels have been treated in a number of theoretical investigations, most of which are confined to panels of infinite aspect ratio. (See refs. 1, 2, 3, 9, 11, 23, and 26.) In addition, a few experimental studies have been made. (See refs. 13, 16, and 23.)

The effect of buckling may be seen from the results in figure 5. The required thickness ratio t/a is plotted against Mach number for steel panels of infinite span in air at sea level. Curves are shown at high Mach numbers for both pinned and clamped panels. The upper pair of curves (fig. 5) are calculated boundaries which apply to buckled panels of infinite span without regard to buckle depth. They result from closed-form solutions based on the so-called "transtability" concept. (See ref. 1.) The lower pair of curves (fig. 5) are the corresponding results for flat panels. Note that the buckled panels require much larger thicknesses to prevent flutter than do the unbuckled panels.

Experimental results for clamped buckled panels are also shown in figure 5. With the exception of the point at $M = 2.18$, these results were taken from reference 16. (They have been corrected to apply to steel at sea level.) The point at $M = 2.18$, which was obtained independently (ref. 23), appears to confirm the other experimental results. The dashed theoretical curve for clamped buckled panels differs on the conservative side by about 12 percent.

In references 13 and 16, some experimental results were presented for a group of clamped rectangular panels buckled by heating. These results are repeated in figure 6.

Figure 6 is constructed with modified-thickness-ratio parameters for both the ordinate and abscissa. One thickness ratio is based on the chord and the other, on the span. The data are applicable to all Mach numbers greater than about 1.3. The construction of the plot is such that data for panels of a given aspect ratio fall on a radial line from the origin. The aspect ratios and orientation of the test panels are shown by the small figures at the outer ends of the radial lines. In each case, node lines are shown to illustrate the observed buckle patterns. The distance outward from the origin of the plot is a measure of the panel thickness. Solid points designate panels that fluttered and open points indicate that no flutter was observed.

An attempt has been made to calculate a theoretical flutter boundary for this particular group of panels buckled by heating. Because the midplane stresses in the test panels had not been measured, it was necessary to estimate these stresses from the observed buckle patterns. Some details of this calculation are given in the appendix. The solid curve is the resulting approximate boundary. The curve has been shown dashed on the lower right because the calculation is a two-mode approximation which must be considered unreliable for the very low-aspect-ratio panels.

The agreement in figure 6 between the theoretical boundary and the experimental data encourages the viewpoint that successful analyses of rectangular buckled panels are possible. Of particular interest is the finding that, as in the case of flat rectangular panels, there are critical combinations of aspect ratio and stress for which even thick panels can flutter. One such critical point occurs in the calculated boundary near an aspect ratio of 0.5. Perhaps this explains why the test panels with an aspect ratio of 0.5 were more prone to flutter than the other test panels.

It must be pointed out that the theoretical boundary is quite sensitive to the type of buckling load, especially for aspect ratios between 0.5 and 2. Consequently, this boundary applies only to the particular series of tests shown. It should be emphasized that this boundary is not intended to be a universal flutter boundary for finite-aspect-ratio buckled panels. In fact, no such universal boundary exists.

CIRCULAR CYLINDERS

One other configuration which has received some preliminary attention is the infinitely long thin-walled circular cylinder with axial flow over its outer surface. (See refs. 14, 21, and 27.) Typical results, for empty steel cylinders, are shown in figure 7.

The ordinate in figure 7 is the ratio of wall thickness to cylinder radius t/r and the abscissa is the Mach number. The curves are theoretical flutter boundaries which apply to unstiffened infinitely long cylinders. The solid curves are boundaries that result when there is no static-pressure differential across the wall; the dashed curves, on the other hand, correspond to the existence of enough internal pressure to cause a circumferential tension near the ultimate.

It will be noted that the required wall thicknesses of unstiffened infinitely long cylinders increase rapidly with increase in Mach number and very quickly reach prohibitive values, even at very high altitudes. Internal pressure is seen to produce only small percentage reductions in these large thicknesses. Furthermore, it has been pointed out in reference 27 that negligible benefit can be expected if internal damping is taken into account.

On the other hand, an effect that shows promise of reducing the thicknesses to reasonable values is the effect of finite length. For the purpose of illustrating this effect, an approximate calculation has been made for a steel cylinder with rigid ring-stiffeners spaced a radius apart. The calculation is based on equation (42) of reference 21 with the air forces replaced, for simplicity, by their plane static approximation. This simplification imposes the requirements that the Mach number M be large and that $M \frac{r}{a} \gg \frac{n}{2\pi}$ where n is the number of full waves around the circumference of the cylinder (corresponding to the maximum required thickness for the prevention of flutter) and a is the distance between stiffeners. The resulting thickness ratio required to prevent flutter at $M = 6$ of the stiffened steel cylinder at an altitude of 35,000 feet with no internal pressure is shown by the small circle in figure 7. This result is only approximate because the above conditions are only approximately satisfied (maximum t/R corresponds to $n = 26$). However, it has the correct order of magnitude. Furthermore, in contrast to the extremely large thickness ratios predicted at $M = 6$ for the unstiffened infinitely long cylinder, the result for the stiffened cylinder has a reasonable order of magnitude. Thus, the large thicknesses predicted by analyses of infinitely long unstiffened cylinders apparently do not apply to practical configurations of finite length.

CONCLUDING REMARKS

It may be stated that much progress has been made in the solution and understanding of the panel flutter problem. Although, in general, panel flutter is of concern only in the design of very thin panels, there appear to be critical combinations of panel aspect ratio and midplane stress for which even very thick panels may flutter. It should be emphasized that there is a need for more work on certain phases of the panel flutter problem. In particular, there is a need for further consideration of cylinders of finite length and for more experimental results for assessing the validity of theoretical analyses.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 7, 1957.

APPENDIX

TRANSTABILITY ANALYSIS OF THE FLUTTER OF BUCKLED

RECTANGULAR PANELS WITH CLAMPED EDGES

The application of the "transtability" concept (ref. 1) to rectangular buckled panels with pinned edges is discussed in reference 26. The corresponding results, for clamped panels, are presented in this appendix.

The transtability problem is governed by the plate equation

$$D\nabla^4 w + P_x w_{xx} + P_y w_{yy} + \frac{2q}{\sqrt{M^2 - 1}} w_x = 0 \tag{1}$$

where w is the panel deflection and the subscripts denote differentiation, and by the proper boundary conditions. The assumption is made that the panel deflection shape is adequately represented in the chordwise direction by a linear combination of the first two modes of a uniform clamped-clamped beam and in the spanwise direction by a single mode, the pth mode. Then, by the Galerkin method, the condition for the existence of a nontrivial solution is found to be

$$I_{12}^{(1)} I_{21}^{(1)} \lambda^2 = \left(I_{11}^{(4)} + A_p I_{11}^{(2)} + B_p\right)\left(I_{22}^{(4)} + A_p I_{22}^{(2)} + B_p\right) \tag{2}$$

where

$$\left.\begin{aligned}
\lambda &= \frac{2qa^3}{\sqrt{M^2 - 1}\, D} \\
A_p &= \frac{P_x a^2}{D} + 2\left(\frac{a}{b}\right)^2 I_{pp}^{(2)} \\
B_p &= \left(\frac{a}{b}\right)^2 I_{pp}^{(2)} \left[\frac{P_y a^2}{D} + \left(\frac{a}{b}\right)^2 \frac{I_{pp}^{(4)}}{I_{pp}^{(2)}}\right] \\
I_{mj}^{(i)} &= \int_0^1 \phi_m(\xi) \frac{d^i}{d\xi^i}\left[\phi_j(\xi)\right] d\xi
\end{aligned}\right\} \tag{3}$$

The function $\phi_m(\xi)$ is the mth mode for a beam of unit length. The beam modes and the integrals $I_{mj}^{(i)}$ are given conveniently in references 29 and 30.

As pointed out in reference 26, the critical value of the dynamic-pressure parameter λ depends on the midplane loads P_x and P_y applied to the panel at the boundaries or induced by heating a panel with fixed boundaries. Let $P_y = RP_x$. Substituting A_p and B_p from equations (3) into equation (2) yields a quadratic equation for the buckling loads P_x corresponding to given values of R and λ. As λ increases from zero, the buckling loads approach each other until at the critical "transtability" value of λ they coalesce and disappear. This condition is given by the vanishing of the discriminant of the quadratic which yields the result

$$\lambda_{cr} = \frac{\left\{\left[I_{22}^{(4)} + 2\left(\frac{a}{b}\right)^2 I_{pp}^{(2)} I_{22}^{(2)} + \left(\frac{a}{b}\right)^4 I_{pp}^{(4)}\right]\left[1 + \frac{I_{pp}^{(2)}}{I_{11}^{(2)}}\left(\frac{a}{b}\right)^2 R\right] - \left[I_{11}^{(4)} + 2\left(\frac{a}{b}\right)^2 I_{pp}^{(2)} I_{11}^{(2)} + \left(\frac{a}{b}\right)^4 I_{pp}^{(4)}\right]\left[1 + \frac{I_{pp}^{(2)}}{I_{22}^{(2)}}\left(\frac{a}{b}\right)^2 R\right]\frac{I_{22}^{(2)}}{I_{11}^{(2)}}\right\}}{2I_{12}^{(1)}\sqrt{\left[1 + \frac{I_{pp}^{(2)}}{I_{11}^{(2)}}\left(\frac{a}{b}\right)^2 R\right]\left[1 + \frac{I_{pp}^{(2)}}{I_{22}^{(2)}}\left(\frac{a}{b}\right)^2 R\right]\frac{I_{22}^{(2)}}{I_{11}^{(2)}}}} \tag{4}$$

Calculations have been made of the quantities:

$$\left(\sqrt{M^2 - 1}\,\frac{E}{q}\right)^{1/3}\frac{t}{a} = \left[\frac{24\left(1 - \mu^2\right)}{\lambda_{cr}}\right]^{1/3}$$

and

$$\left(\sqrt{M^2 - 1}\,\frac{E}{q}\right)^{1/3}\frac{t}{b} = \left[\frac{24\left(1 - \mu^2\right)}{\lambda_{cr}}\right]^{1/3}\frac{a}{b}$$

for a series of test panels buckled by heating. The integer p, specifying the number of half-waves in the spanwise direction, was taken as the number of spanwise half-waves in the observed buckle patterns.

Furthermore, since the actual loads in the test panels had not been measured, the ratio $R = \frac{P_y}{P_x}$ was estimated from the buckle patterns with the aid of figure 5 of reference 31. Specifically, for each aspect ratio, the average longitudinal buckling load corresponding to the number of observed spanwise or chordwise half-waves was used along with the corresponding transverse load. For an aspect ratio of 1, it was assumed that $R = 1$ $(P_x = P_y)$. The resulting calculated boundary is shown in figure 6 along with the test results.

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REQUIRED THICKNESS OF INFINITE-ASPECT-RATIO UNSTRESSED STEEL PANELS AT SEA LEVEL

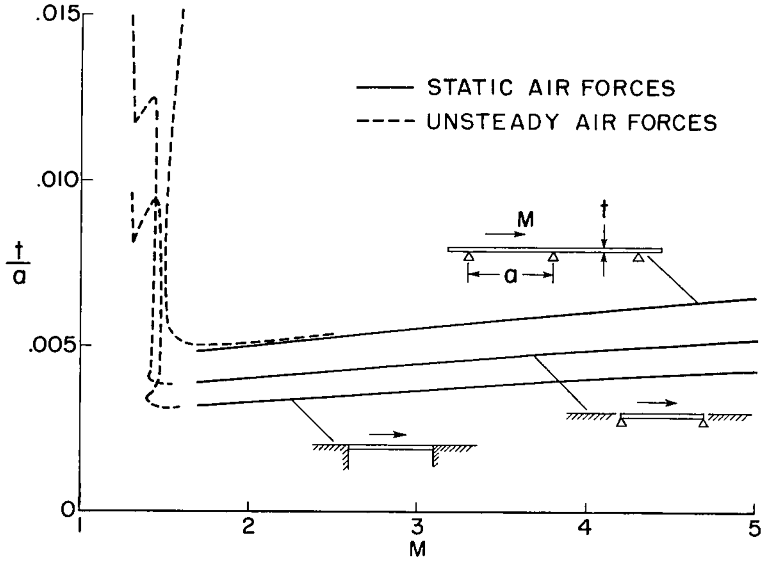

Figure 1

EFFECT OF ASPECT RATIO ON REQUIRED THICKNESS OF UNSTRESSED SIMPLY-SUPPORTED STEEL PANELS AT SEA LEVEL

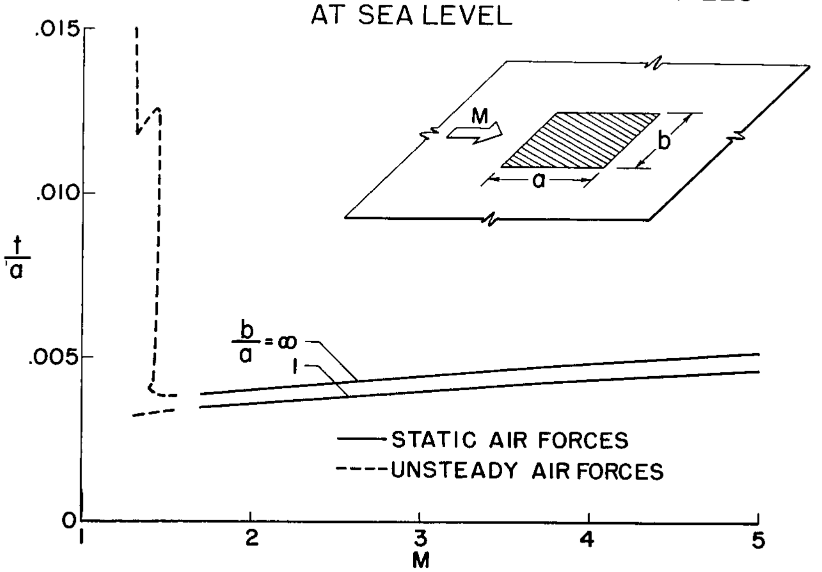

Figure 2

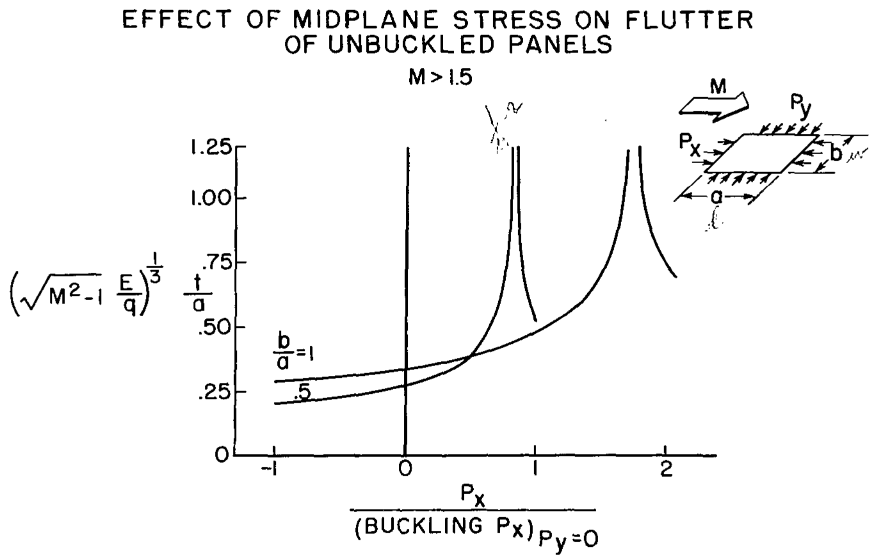

Figure 3

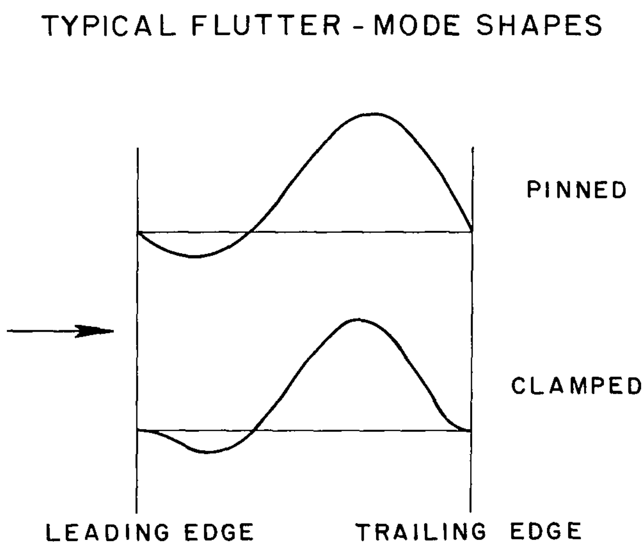

Figure 4

EFFECT OF BUCKLING ON REQUIRED THICKNESS OF INFINITE - ASPECT - RATIO STEEL PANELS AT SEA LEVEL

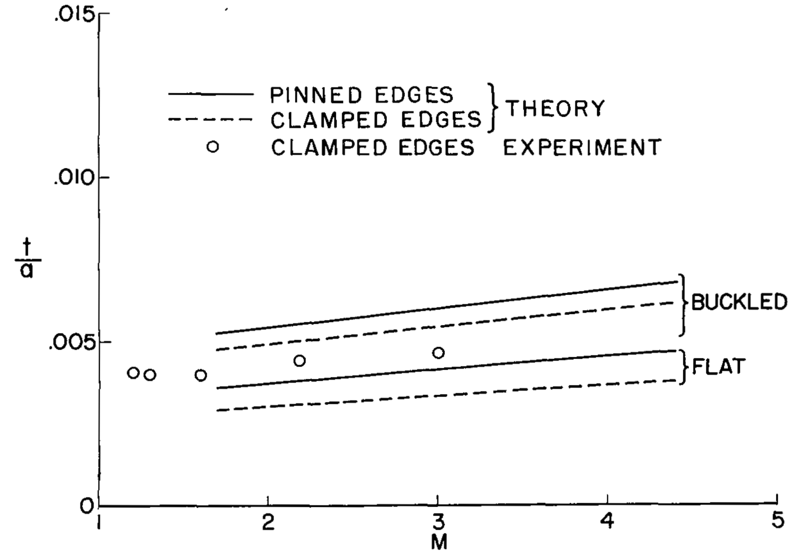

Figure 5

REQUIRED THICKNESSES OF A SERIES OF TEST PANELS BUCKLED BY HEATING

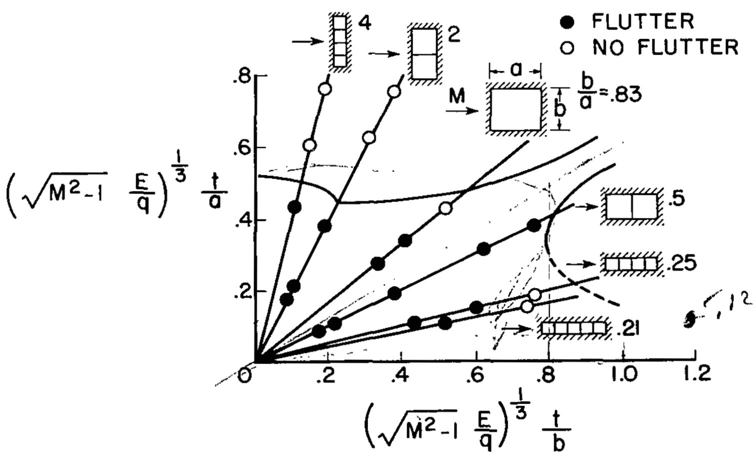

Figure 6

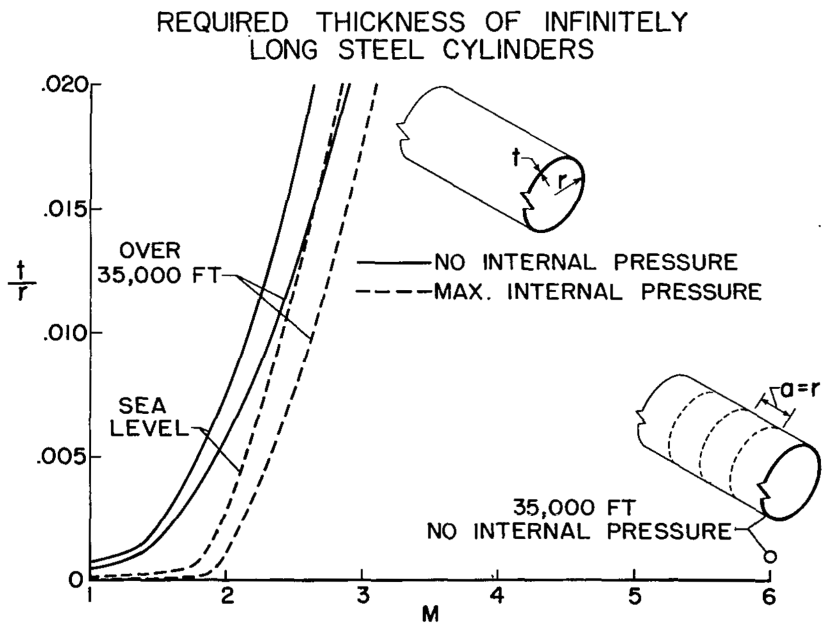

Figure 7

NACA - Langley Field, Va.